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1) E^2 de $\{x_1, x_2\}$ koordinat sistemi verilsin.

$$X = 2x_2^2 \frac{\partial}{\partial x_1} + \left(x_1^2 - x_2\right)\frac{\partial}{\partial x_2}$$

vektör alanı ve $P = (-1,3) \in E^2$ için $X(P) = \overrightarrow{X_P}$ tanjant vektörünü bulunuz ve şeklini çiziniz.

2) E^3 de $\{x_1, x_2, x_3\}$ koordinat sistemi verilsin.

$$X = x_2^2 \frac{\partial}{\partial x_1} - x_1 \frac{\partial}{\partial x_3},\quad Y = x_1 \frac{\partial}{\partial x_1} - x_1 x_2 \frac{\partial}{\partial x_2} - x_3 \frac{\partial}{\partial x_3} \in \chi\left(E^3\right)$$

ve $f = x_1 x_3$ olsun. O halde $P = (1,0,1) \in E^3$ için $\left(Y\left(X\left(f\right)\right)\right)(P) = ?$

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CEVAP ANAHTARI

1) $X(P) = \left(2x_2^2 \frac{\partial}{\partial x_1} + (x_1^2 - x_2)\frac{\partial}{\partial x_2}\right)(P)$

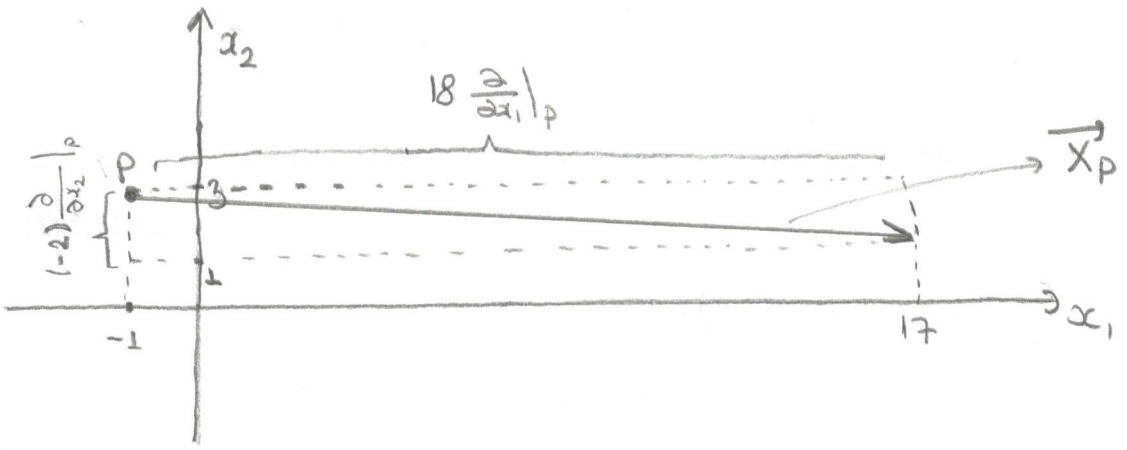
$= 2x_2^2(P) \left.\frac{\partial}{\partial x_1}\right|_P + (x_1^2(P) - x_2(P)) \left.\frac{\partial}{\partial x_2}\right|_P$

olur. $P = (-1,3) \in E^2$ için

$X(P) = \vec{X}_P = 2.(3)^2 \left.\frac{\partial}{\partial x_1}\right|_P + ((-1)^2 - 3) \left.\frac{\partial}{\partial x_2}\right|_P$

$= 18 \left.\frac{\partial}{\partial x_1}\right|_P + (-2) \left.\frac{\partial}{\partial x_2}\right|_P$

bulunur. $\vec{X}_P$ nin şeklini çizelim.



2) $Y(X(f)) = Y\left(\left(x_2^2\frac{\partial}{\partial x_1} - x_1\frac{\partial}{\partial x_3}\right)(x_1x_3)\right)$

$= Y\left(x_2^2\frac{\partial}{\partial x_1}(x_1x_3) - x_1\frac{\partial}{\partial x_3}(x_1x_3)\right)$

$= Y\left(x_2^2\frac{\partial(x_1x_3)}{\partial x_1} - x_1\frac{\partial(x_1x_3)}{\partial x_3}\right)$

$= Y\left(x_2^2\cdot x_3 - x_1^2\right)$

$= \left(x_1\frac{\partial}{\partial x_1} - x_1x_2\frac{\partial}{\partial x_2} - x_3\frac{\partial}{\partial x_3}\right)\left(x_2^2x_3 - x_1^2\right)$

$= x_1\frac{\partial}{\partial x_1}(x_2^2x_3 - x_1^2) - x_1x_2\frac{\partial}{\partial x_2}(x_2^2x_3 - x_1) - x_3\frac{\partial}{\partial x_3}(x_2^2x_3 - x_1^2)$

$= x_1\frac{\partial(x_2^2x_3 - x_1^2)}{\partial x_1} - x_1x_2\frac{\partial(x_2^2x_3 - x_1)}{\partial x_2} - x_3\frac{\partial(x_2^2x_3 - x_1^2)}{\partial x_3}$

$= -2x_1^2 - 2x_2^2x_1x_3 - x_3x_2^2$

bulunur. Böylece $P = (1,0,1)$ olmak üzere

$(Y(X(f)))(P) = (-2x_1^2 - 2x_2^2x_1x_3 - x_3x_2^2)(P)$

$= -2(x_1(P))^2 - 2(x_2(P))^2\cdot x_1(P)\cdot x_3(P) - x_3(P)(x_2(P))^2$

$= -2 - 2\cdot 0\cdot 1\cdot 1 - 1\cdot 0$

$= -2$

elde edilir.